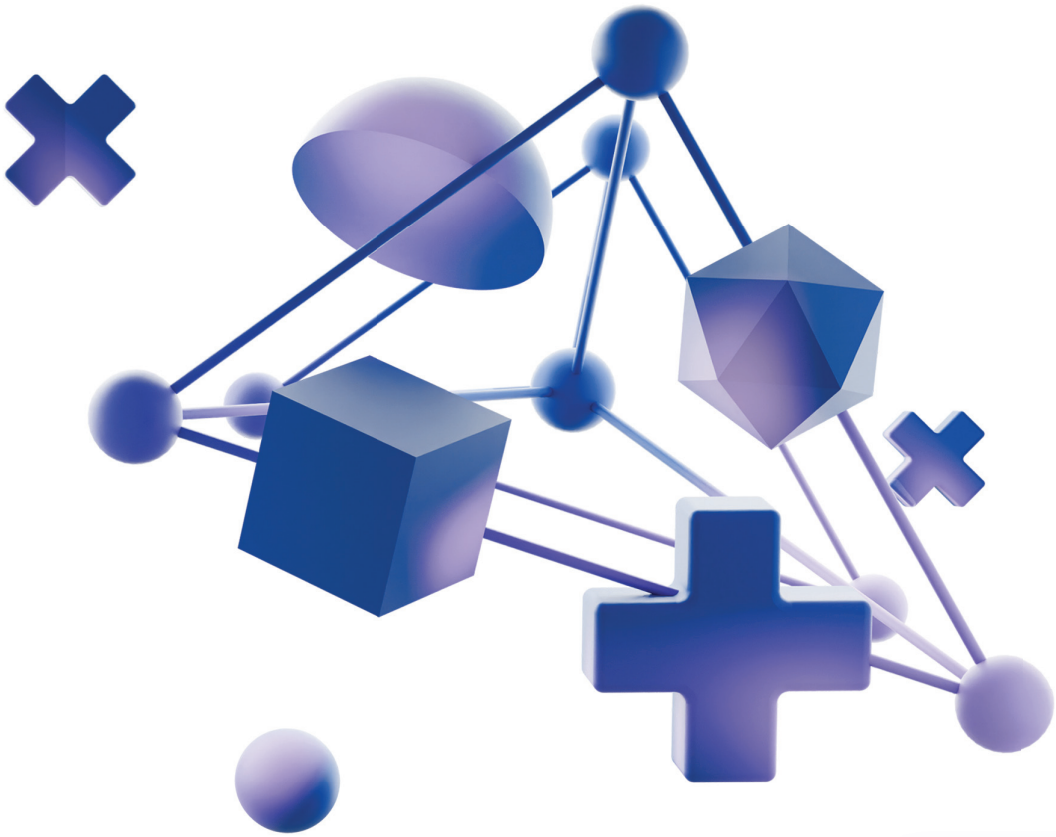
MASTERING MATH



TEACHER EDITION

BLUE LEVEL

Editorial Project Manager:
Rabbi Levi Friedman

Editor:
Miriam Shulamis Eisemann

Project Director:
Rabbi Mordechai Resnick

Creative Director:
Glenna Daniel

Chief Curriculum Developer:
Chumy Spiegel

Graphic Designer:
Miri Perry

Author:
Miriam Olsberg

Contributing Authors:
Tamar Ginges
Rivki Steinhaus

Printed in the USA in August 2025.

ISBN: 978-1-968294-49-6

Achievements
Educational Services

1072 Madison Ave.
Lakewood, NJ 08701

www.achievementsES.com
info@achievementsES.com
800-742-1803

Table of Contents

Table of Contents

Introduction for the Teacher

CONTENTS:

A. PURPOSE OF THE COURSE

The Achievements Mastering Math course was created by a team of professional educators, writers, and curriculum developers. It was built from the very start with the goal of presenting the material in a methodical and organized manner as well as constant review of the basic operations to help the students attain a mastery of each math skill. This course is aligned with the latest state standards, including Common Core and NGMLS and is intended to prepare middle school students for high school math by reviewing all the foundational skills as well as gaining a solid understanding of pre-algebra concepts.

MATHEMATICS

Mathematics is the language of numbers. Describing the world around us using numbers helps us measure, quantify, compute, compare, describe, solve, and predict many common phenomena around us. For example, someone may claim that it is very cold outside today, but exactly how cold is it? Did the temperature rise, fall, or stay the same in the last two hours? Without a precise measuring system that ascribes numbers to small variations in temperature, these questions would be very hard to answer. Manipulating these numbers can help us track changes, compare measurements, and solve problems. Mathematics is considered a core curriculum in middle school due to the frequency of its use in real life.

Although most math courses typically focus on how to solve number problems, and students spend most of their time mastering math skills through rote, they must also be able to interpret and solve problems that are not presented in typical math lingo. They must learn to apply math concepts to everyday situations and determine which math operation or method is required to find a solution.

Middle school mathematics covers many number skills, including:

- Number theory
- Arithmetic using the four operations
- Powers, roots, and scientific notation
- Fractions, decimals, percents, and ratios
- Interpreting data and reading charts
- Coordinate graphing and functions
- Algebraic expressions
- Geometry

SUGGESTIONS FOR THE TEACHER

- Teachers are encouraged not to allow calculators. Throughout the course, great emphasis is placed on computing the four basic operations by hand to help students gain greater mastery of the skills through rote.
- Students should be required to show all algebraic work, even with simple examples that can be computed in their head. The reason for this is twofold. First, it helps them understand what they are doing so they can apply the same concepts to harder examples. Secondly, it helps them identify errors that they may overlook while using mental math.

B. COMMON CORE AND NGMLS STANDARDS

Common Core

The following Common Core standards for grade 8 are covered in this course:

THE NUMBER SYSTEM

CCSS.MATH.CONTENT.6.NS.A.1
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

CCSS.MATH.CONTENT.6.NS.A.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π2).

EXPRESSIONS AND EQUATIONS

CCSS.MATH.CONTENT.6.EE.A.1
Know and apply the properties of integer exponents to generate equivalent numerical expressions.

CCSS.MATH.CONTENT.6.EE.A.2
Use square root and cube root symbols to represent solutions to equations of the form x2 = p and x3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.

CCSS.MATH.CONTENT.6.EE.A.3
Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

CCSS.MATH.CONTENT.6.EE.A.4
Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.

CCSS.MATH.CONTENT.6.EE.B.5
Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

CCSS.MATH.CONTENT.6.EE.B.6
Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

CCSS.MATH.CONTENT.6.EE.C.7
Solve linear equations in one variable.

CCSS.MATH.CONTENT.6.EE.C.7.A
Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation results.

CCSS.MATH.CONTENT.6.EE.C.7.B
Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

FUNCTIONS

CCSS.MATH.CONTENT.6.F.A.1
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

CCSS.MATH.CONTENT.6.F.A.2
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

CCSS.MATH.CONTENT.6.F.A.3
Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

CCSS.MATH.CONTENT.6.F.B.4
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

CCSS.MATH.CONTENT.6.F.B.5
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

GEOMETRY

CCSS.MATH.CONTENT.6.G.A.1
Verify experimentally the properties of rotations, reflections, and translations.

CCSS.MATH.CONTENT.6.G.A.1.A
Lines are taken to lines, and line segments to line segments of the same length.

CCSS.MATH.CONTENT.6.G.A.1.B
Angles are taken to angles of the same measure.

CCSS.MATH.CONTENT.6.G.A.1.C

Parallel lines are taken to parallel lines.

CCSS.MATH.CONTENT.6.G.A.2
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

CCSS.MATH.CONTENT.6.G.A.3
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

CCSS.MATH.CONTENT.6.G.A.4
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

CCSS.MATH.CONTENT.6.G.A.5
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

CCSS.MATH.CONTENT.6.G.B.6
Explain a proof of the Pythagorean Theorem and its converse.

CCSS.MATH.CONTENT.6.G.B.7
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

CCSS.MATH.CONTENT.6.G.C.9
Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

NGMLS

Next Generation Mathematics Learning Skills (NGMLS) is a newly developed system implemented in New York State that largely follows the Common Core standards, with some minor changes. Because most of the standards are identical and equally correlate to this course, we will suffice by providing the state's list of changes to the standards: www.nysed.gov/sites/default/files/programs/curriculum-instruction/nys-math-standards-grade-8-snapshot.pdf

C. HOW TO USE THIS BOOK

The following is an explanation of the various features of the math lessons.

A. Leader Image

The leader image will typically tie into one of the word examples at the end of the lesson. Its main purpose is to create a more stimulating and colorful environment for the student as well as highlight the idea that math is all around us.

B. Explanation

The lesson starts with one or several paragraphs explaining in words the method or procedure covered in the lesson. Math words are defined in this section and colored blue. Various math symbols are often introduced here as well.

C. Demonstration

This section immediately follows the explanation and is marked with the word "Example" and surrounded by a shaded box. It provides a demonstration of how the math is performed. The example is solved step by step to ensure that the entire process is clearly described to the student.

D. Check-In

This section is purposely designed to be highly visible to the student with a distinct icon and large heading. Teachers are encouraged to complete the Check-In question together with the class. It is a transitional problem between the demonstration and independent practice and gives students a chance to clarify anything they still do not understand. Answers are provided in teacher's edition only.

E. Key Points

Before beginning independent practice, the key points of the lesson are quickly reviewed. Read these points with the class and encourage students to raise any questions they may have.

F. Practice

Practice questions should be completed by students as independent practice. No calculators should be allowed. Answers are provided in teacher's edition and can be reviewed section by section. The lesson will end off with several word problems. Students should be able to identify the method required to solve the problem on their own.

G. Chapter Review

This section appears at the end of every chapter. It should be completed as a class review of everything learned throughout the chapter. Following this review, students should receive home review materials and should be notified when they will be tested on the chapter.

When using the teacher's edition, bear in mind that anything that appears in red is only available to the teacher. All black text is visible to the student as well.

D. USB EXTERNAL MATERIALS

The following supplemental materials are provided on the course USB in digital form (PDF):

- Homework booklet
- Chapter summary
- Chapter review
- Chapter test

CUMULATIVE REVIEWS

All of the above, except the chapter summary, come in two versions, student and teacher.

1. Homework Booklet

The homework booklet should be printed out for each student at the start of each chapter, and the teacher should retain a copy of the teacher version as well. Students are to complete the homework for each lesson that night, and answers should be reviewed the next day. These booklets may then be collected at the end of the chapter and spot-checked to ensure that they were completed properly. Students should be graded for completion of the work, not for the accuracy of their answers.

2. Chapter Summary

Once a chapter is completed in class, students should receive the chapter summary and review. The summary is a quick reminder of the main points of each lesson. It is purposely written in shorthand. The teacher may review it quickly in class as well.

3. Chapter Review

This is similar to the chapter review that appears in the book and is intended as an aid for students to study for the chapter test. It should not be handed in or graded; rather, students should use the examples as a way to practice for the test. The teacher may even choose to provide students with the correct answers so that they can check their results as they go along.

4. Chapter Test

The student version should be printed out and distributed to each student on the day of the test. Completed tests should be collected and then graded using the teacher version.

5. Cumulative Reviews

The cumulative reviews reinforce older lessons periodically and appear after the third, sixth, and final chapters. Each one tests on all previous chapters. These may be completed in class, for homework, or graded as a quiz.

Whole Numbers and Equations

IN THIS CHAPTER

Place Value

YOU WILL LEARN how to find the place value of a digit in a number.

Space math uses big numbers, and understanding place value helps us make sense of the universe. NASA tracks the distance from Earth to other planets. The average distance from Earth to Pluto is about 3,224,000,000 miles.

This number is written in standard form; what do the digits mean?

Example

Use the chart below to identify the place and value of the 3 in 3,224,000,000.

TRILLIONS			BILLIONS			MILLIONS			THOUSANDS			ONES		
hundred trillions	ten trillions	trillions	hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
					3,	2	2	4,	0	0	0,	0	0	0

Each group of three numbers is called a period.

The 3 is in the billions place.

Its value is 3,000,000,000.

Numbers can be written in different ways.

The budget for a new satellite mission is $54,260,000.

You can write this number in different forms.

- **Standard form:** 54,260,000
- **Word form:** fifty-four million, two hundred sixty thousand
- **Short-word form:** 54 million, 260 thousand
- **Expanded form:** 50,000,000 + 4,000,000 + 200,000 + 60,000

WHAT DO YOU THINK?

▶ How do commas help you read and write numbers?

They separate between hundreds, thousands, and millions.

▶ Although zero has no value, it is very important. Explain the difference between 25 and 2,005.

2,005 is much larger than 25. The zeros hold the place of the tens and hundreds, so the 2 is in the thousands place.

TRY IT

Write the place and value of the underlined digit.

1. 15,762,900

Place of the 6: ten thousands

Value of the 6: 60,000

2. 4,098,000

Place of the 4: millions

Value of the 4: 4,000,000

PRACTICE

Write the place and the value of the underlined digit.

3. 85,462,020

Place: thousands

Value: 2,000

4. 149,001

Place: ten thousands

Value: 40,000

5. 22,195,000,000

Place: billions

Value: 2,000,000,000

6. 230,920,066

Place: ten millions

Value: 30,000,000

7. 740,884

Place: hundred thousands

Value: 700,000

Write each number in word form.

8. 63,450,000

Sixty-three million, four hundred fifty thousand

9. 11,230,000,000

Eleven billion, two hundred thirty million

10. 232,890,610,000

Two hundred thirty-two billion, eight hundred ninety million, six hundred ten thousand

Write each number in short word form.

11. 40,789,000,000

40 billion, 789 million

12. 26,845,951,000,000

26 trillion, 845 billion, 951 million

13. 9,012,000

9 million, 12 thousand

Write each number in expanded form.

14. 45,566,000

40,000,000 + 5,000,000 + 500,000 + 60,000 + 6,000

15. 701,592,608

700,000,000 + 1,000,000 + 500,000 + 90,000 + 2,000 + 600 + 8

16. 84,005,037,700

80,000,000,000 + 4,000,000,000 + 5,000,000 + 30,000 + 7,000 + 700

Write each number in standard form.

17. 230 million, 825 thousand

230,825,000

18. Sixty-five trillion, eighty million, two hundred ten thousand

65,000,080,210,000

19. 800,000,000 + 70,000,000 + 40,000 + 1,000 + 600 + 20

870,041,620

20. 569 trillion, 60 billion, 400 million, 788 thousand

569,060,400,788,000

21. Ninety-six million, six hundred forty-three thousand

96,643,000

22. 40,000,000 + 5,000,000 + 300,000 + 7,000 + 800 + 50 +5

45,307,855

23. 48 billion, 600 million, 34 thousand

48,600,034,000

24. Twenty-two billion, seven hundred sixty-one thousand

22,000,761,000

25. 3,000,000,000 + 500,000,000 + 90,000,000 + 10,000 + 4,000 + 200 + 30

3,590,014,230

26. 60,000,000 + 6,000,000 + 900,000 + 80,000

66,980,000

REAL–WORLD PROBLEM SOLVING

NASA tracks the average distance from Earth to different planets in our solar system. The table below shows these distances. Use the table to answer the questions.

PLANET	AVERAGE DISTANCE FROM EARTH (KM)
Mercury	91,000,000
Venus	42,000,000
Mars	225,000,000
Jupiter	778,000,000
Saturn	1,430,000,000

27. What is the value of the 4 in the distance to Venus?

40,000,000

28. Write the distance to Saturn in expanded form.

1,000,000,000 + 400,000,000 + 30,000,000

29. Which planet has a 7 in the hundred millions place?

Jupiter

FUN FACT

There are more stars in the universe than grains of sand on Earth.